

Prof. David Draper  
Department of Statistics  
University of California, Santa Cruz

## AMS 131: Quiz 7

Name: \_\_\_\_\_

All parts of this problem are unrelated (i.e., the assumptions in part (x) apply only to part (x)). All expectations and variances are assumed to exist and to be finite.

- (a) You're working with two random variables  $X$  and  $Y$ , which may be dependent and for which  $V(X) = V(Y)$ . Show that the random variables  $W = (X+Y)$  and  $Z = (X-Y)$  are uncorrelated. *Hint:* Nothing fancy — just simplify the covariance of  $W$  and  $Z$ , using properties of covariance discussed in class and discussion section.
- (b) You're working with two random variables  $X$  and  $Y$  that are negatively correlated. Which is bigger —  $V(X + Y)$  or  $V(X - Y)$  — or are they equal? Show your calculations.
- (c) You're working with two random variables  $X$  and  $Y$  such that  $V(X) = 9$ ,  $V(Y) = 4$ , and  $\rho(X, Y) = -\frac{1}{6}$ . Compute  $V(X + Y)$  and  $V(X - Y)$  (show your calculations).
- (d) You and your research assistant (RA) are working with two random variables  $X$  and  $Y$ , and your RA has computed the following values:  $E(X) = 3$ ,  $E(Y) = 2$ ,  $E(X^2) = 10$ ,  $E(Y^2) = 29$ , and  $E(XY) = 0$ . Show that there must be something wrong in this computation. *Hint:* Consider the bounds on variances and correlations.