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AMS 131: Quiz 5

Name: _____

You're working on a problem involving two continuous random variables X and Y, and you figure out that their joint PDF has the following form:

$$f_{X,Y}(x,y) = \left\{ \begin{array}{cc} c\,x^2 & \text{for } 0 \le y \le 1 - x^2 \\ 0 & \text{otherwise} \end{array} \right\} . \tag{1}$$

(a) Sketch the support S of this bivariate distribution.

(b) Compute the normalizing constant c.

(c) It can be shown that the marginal PDFs of X and Y with this joint PDF are

$$f_X(x) = \left\{ \begin{array}{cc} \frac{15}{4} x^2 (1 - x^2) & \text{for } -1 \le x \le 1 \\ 0 & \text{otherwise} \end{array} \right\} \tag{2}$$

and

$$f_Y(y) = \left\{ \begin{array}{cc} \frac{5}{2} (1 - y)^{\frac{3}{2}} & \text{for } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{array} \right\} . \tag{3}$$

Verify that both of these marginals are correct.

(d) Are X and Y independent in this joint distribution? Explain briefly.