AMS131
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doc. cam. notes ①

~~this last. of time: functions of rvs, expectation~~

read: DS ch. 4 pp. 215–260

~~next~~ this time: expectation of sums and products, variance, SD; other moments of the dist.

Case study: option pricing

Silicon valley companies give signing bonuses as incentives to accept their job offers. These are often in the form of stock options: an opportunity to buy N shares of the company one year from now at a (known) price, S. If in your view the stock is likely to rise over the next year, you'll be able to sell at a profit. Define $X = \left(\begin{array}{c}\text{price of the}\\ \text{stock 1 year}\\ \text{from now}\end{array}\right)$.

For simplicity pretend X is discrete

with only 2 values: $x_1 < S$ and $x_2 > S$, let $p = P(X = x_2)$, the prob. that the stock will rise in value. You'd like to evaluate these stock options (e.g., to compare one company's job offer with that of another), but (of course) you don't know X. Let Y = value of option for one share at $\$S$ 1 year from now.

If ($X = x_1 < S$), the option is worthless and $Y = 0$; otherwise (ignoring dividends & costs of buying & selling stocks) if ($X = x_2 > S$) then the option is worth $(x_2 - S)$; thus

$$Y = h(X) = \begin{cases} 0 & \text{if } X = x_1 \\ (x_2 - S) & x_2 \end{cases}.$$

To see how valuable the option is, you have to compare it to the return you would have received ③

if you had not "exercised" the stock option;

a reasonable point of comparison would

be to invest in a bond or other fixed security that pays α%/year.

A fair measure of worth of the option

would be the <u>present value</u> of Y,

defined to be the number c such that

$E(Y) = (1+\alpha)\cdot c$. But we already know

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that $E(Y) = 0\cdot(1-p) + (x_2 - S)\cdot p = (x_2 - S)p$,

so $(1+\alpha)\cdot c = (x_2 - S)\cdot p$ and $c = \left(\frac{x_2 - S}{1+\alpha}\right)\cdot p$.

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to finish the calculation you need to

specify p. The standard way to do this

in the financial sector is to assume that (4)
the present value of $\underline{X}$ is equal in
expectation to the current value of the
stock price: i.e., to assume that the
expected value of (buying 1 share &
holding it for a year) = (investing the
same amount of money in the risk-free
alternative) – i.e., $E(\underline{X}) = (1+\alpha) \cdot S$.

But we already know that

$E(\underline{X}) = p \cdot x_2 + (1-p) \cdot x_1 \stackrel{\Delta}{=} (1+\alpha) S$; solving

for p gives $p = \dfrac{x_1 - (1+\alpha) S}{x_1 - x_2} = \dfrac{(1+\alpha) S - x_1}{x_2 - x_1}$.

So the fair price (c) of an option to buy one share is given by $c = \left(\frac{x_2 - S}{1 + \alpha} \right) \left[\frac{(1+\alpha) S - x_1}{x_2 - x_1} \right]$. (5)

DS use as illustration
$S = \$200$
$x_1 = \$180$
$x_2 = \$260$
$\alpha = .04$

downside $20 (−10%)

upside $60 (+30%)

realistic in 2001 or so but not today: $\alpha \doteq .01$ now or .02

With these values $c \doteq \$20.19$ (about 10% of the current value of the stock). c is called the risk-neutral price of the option; under the assumptions made here, you could now sell the option today (if you had it) at a fair price of about $20; this would make you an options trader.

An investment that allows people to buy or sell an option on a security (e.g. stock) is called a derivative.