Discussion Section 4

AMS131, 8 Aug 19

①

Y1 = you initially pick door 1

Y2 = 2

Y3 = 3

M1 = MH shows you a goat behind door 1 ; M2, M3

C1 = car really is behind door 1

C2, C3

unknown: (truth) where car is (C1, C2, C3)

data: MH shows you goat behind door —

without loss of generality (WLOG)

let's choose Y1 and M2

this is a job for Bayes's Thm.

~~let's use Bayes's Thm. in odds form~~

$$\frac{P(C3 \mid Y1 \text{ and } M2)}{P(C1 \mid Y1, M2)} = \left[\frac{P(C3)}{P(C1)}\right]\left[\frac{P(Y1, M2 \mid C3)}{P(Y1, M2 \mid C1)}\right]$$

(posterior odds ratio in favor of C3 given Y1 & M2) = (prior odds ratio in favor of C3) (Bayes factor in favor of C3 given Y1 & M2)

$P(C3) = P(C1) = \frac{1}{3}$ so $\frac{P(C3)}{P(C1)} = 1$

$P(Y1, M2 \mid C3) = \cancel{\frac{P(Y1, M2, C3)}{P(C3)}}$

?

$P(A \text{ and } B \mid C) = ?$

$= P(A \mid C) \cdot P(B \mid A, C)$

$= P(B \mid C) \cdot P(A \mid B, C)$

$P(A \text{ and } B) =$

$P(A) \cdot P(B \mid A)$

$= P(B) \cdot P(A \mid B)$

$$\frac{P(Y1, M2 \mid C3)}{P(Y1, M2 \mid C1)} = \frac{\cancel{P(Y1 \mid C3)}\, P(M2 \mid Y1, C3)}{\cancel{P(Y1 \mid C1)}\, P(M2 \mid Y1, C1)} = \frac{\cancel{\frac{1}{3}} \cdot 1}{\cancel{\frac{1}{3}} \cdot \frac{1}{2}}$$

$= 2$ to 1 data odds (Bayes factor)

in favor of C3

given Y1 & M2

$$\frac{p(c3 \mid Y1, m2)}{p(c1 \mid Y1, m2)} = 1 \cdot 2 = o_{c3}$$

$$p_{c3} = \frac{o_{c3}}{1 + o_{c3}} = \frac{2}{3}$$

← cond. prob. car is behind door 3 given Y1 & m2

∴ you should switch

A = unknown (truth)

D = data

$$P(A|D) = \frac{P(A)\,P(D|A)}{P(D)}$$

if $P(A) = 0$ then $P(A|D) = 0$ no matter how the data D came out

if $P(A) = 1$

$$P(A|D) = \frac{\cancel{P(A)}^{1}\,P(D|A)}{P(D)}$$

$$= \frac{P(D \text{ and } A)}{P(D)\,\cancel{P(A)}^{1}} = \frac{P(D)}{P(D)}$$

$= 1$ no matter how data comes out

try not to assume $P(A) = 0$ or 1 unless you're ok with never learning that you're wrong

$\underline{Y}$ = death penalty (DP) = $\begin{cases} \text{yes} \\ \text{no} \end{cases}$

X = race of defendant = $\begin{cases} \text{white (DW)} \\ \text{black (DB)} \end{cases}$

ELM ?

$P(DP) = \frac{36}{326} \doteq 11\%$

Y ✓

$P(DP \mid DW) = \frac{19}{160} \doteq 12\%$

$P(DP \mid DB) = \frac{17}{166} \doteq 10\%$

mildly surprising

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obs. study | enemy: bias from confounding factors (CF) Z_1

Z_1 is a CF $\leftrightarrow$ $\begin{cases} Z_1, X \text{ associated } ✓ \\ \text{and } Z_1, Y \text{ associated } ✓ \end{cases}$

race of victim

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how defeat enemy Z_1 ?

hold Z_1 constant in examining relation between X and Y