Discussion Section 10

roulette

AMS 131
29 Aug 19

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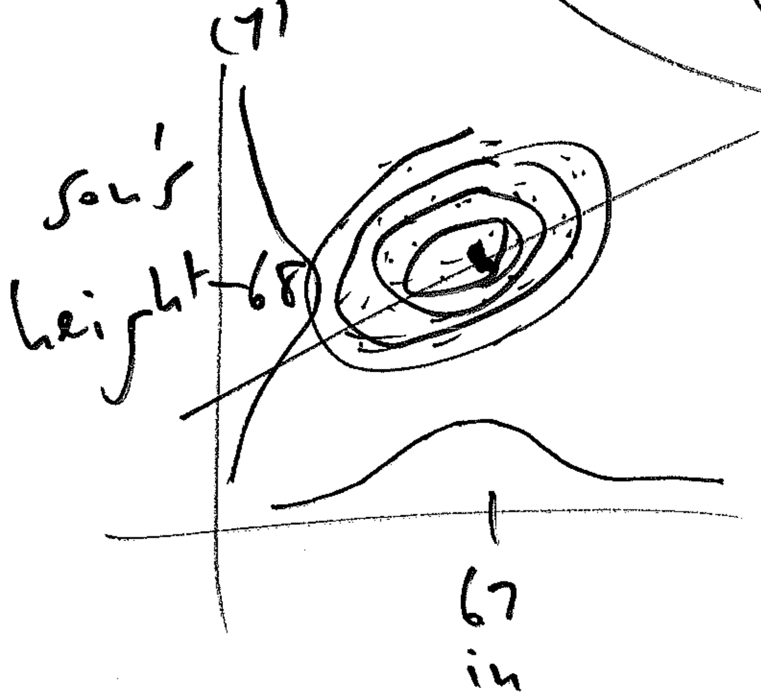
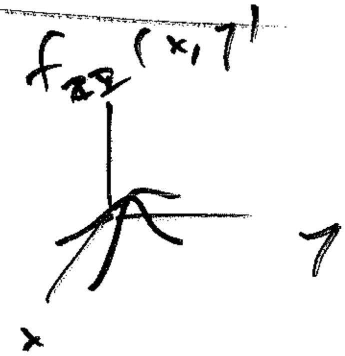
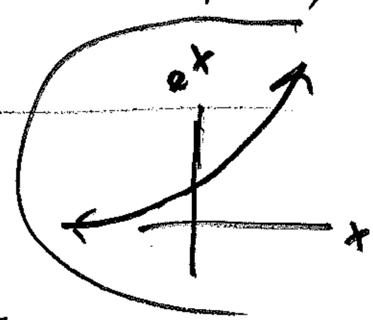
single #, split, ...
(see pp. 2–3)

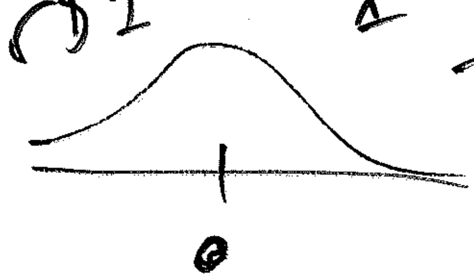




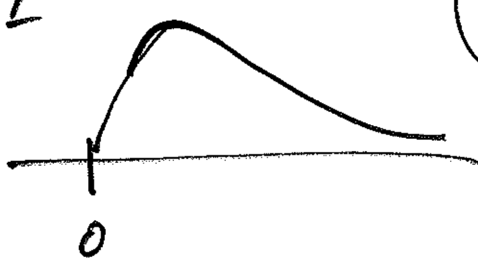
$$r = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s_x^*}\right)\cdot\left(\frac{y_i-\bar{y}}{s_y^*}\right)$$



(1) X



$X \longrightarrow e^X = Y$



roulette single # ②

$$Z_i \overset{IID}{=} \begin{cases} -1 & \text{w.p. } \frac{37}{38} \\ +35 & \frac{1}{38} \end{cases}$$

$$X_i = \frac{Z_i + 1}{36} \overset{IID}{\sim} \text{Bernoulli}\left(\frac{1}{38}\right)$$

$$S = \sum_{i=1}^{n} Z_i = \sum_{i=1}^{n} (36 X_i - 1) = \left(36 \sum_{i=1}^{n} X_i\right) - n$$

$$\sum_{i=1}^{n} X_i \sim \text{Binomial}\left(n, \frac{1}{38}\right)$$

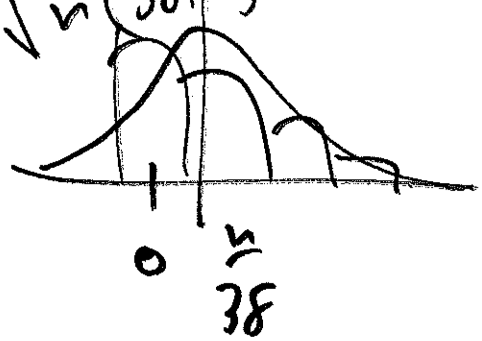
$$(S > 0) \leftrightarrow 36 \sum_{i=1}^{n} X_i > n$$

$$P(S > 0) = P\left(\sum_{i=1}^{n} X_i > \frac{n}{36}\right)$$

$$= 1 - P\left(\sum_{i=1}^{n} X_i \le \frac{n}{36}\right)$$

$n = 35$

SD $\sqrt{n\left(\frac{1}{38}\right)\left(\frac{37}{38}\right)}$



$$Y_i = \begin{cases} -1 & \text{with prob. } 1-p \\ w & p \end{cases}$$

general roulette

$$X_i = \frac{Y_i + 1}{w+1} \sim \text{Bernoulli}(p)$$

$$S = \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} \left[(w+1) X_i - 1 \right] = (w+1)\left(\sum_{i=1}^{n} X_i \right) - n$$

$$\sum_{i=1}^{n} X_i \sim \text{Binomial}(n, p)$$

$$(S > 0) \longleftrightarrow (w+1)\left(\sum_{i=1}^{n} X_i \right) > n$$

$$P(S > 0) = P\left(\sum_{i=1}^{n} X_i > \frac{n}{w+1} \right)$$

$$= 1 - P\left(\sum_{i=1}^{n} X_i \leq \frac{n}{w+1} \right)$$

④

$Y \sim$ Lognormal $(100, 1)$ $\quad Y = e^X$

$X \sim N(100, 1)$

$$B^* = \begin{cases} 50 & p \le \frac{1}{2} \\ 2(p - \frac{1}{2})A & p \ge \frac{1}{2} \end{cases}$$

Bernoulli

$u(x) = 1 + \log(x)$

$$u(x) = x$$

$$f_X(x) = \begin{cases} p & x = A + B \\ 1-p & x = A - B \\ 0 & \text{else} \end{cases}$$

$$E[u(X)] = E(X)$$

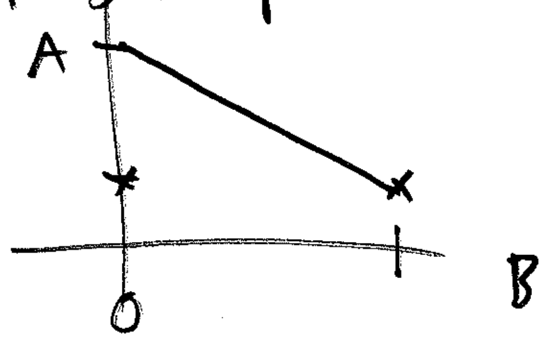
$$= p \cdot (A + B) + (1-p)(A - B)$$

$$= \cancel{pA} + 2pB + A - B \cancel{-pA} + \cancel{pB}$$

$$= A + B(2p - 1)$$

$(p < \frac{1}{2})$

$A + B(2p-1)$

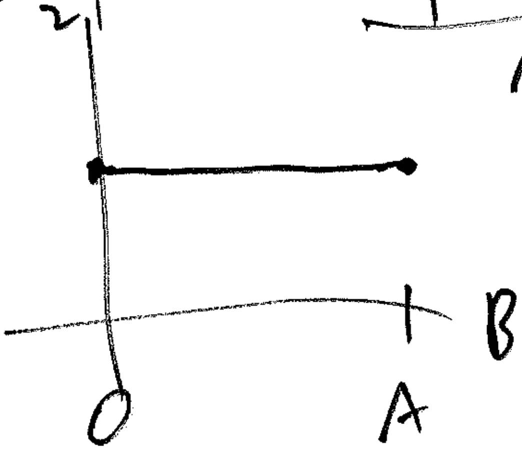


$$B^*_{\text{linear}} = \begin{cases} \$0 \text{ (no bet)} & p < \frac{1}{2} \\ \text{any \$ between \$0 and \$A} & p = \frac{1}{2} \\ \$A \text{ (bet it all)} & p > \frac{1}{2} \end{cases}$$

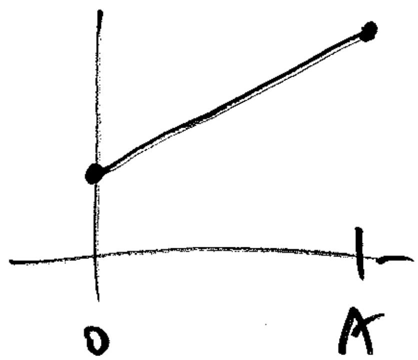
riskier

$A + B(2p-1)$

$(p = \frac{1}{2})$



$p > \frac{1}{2}$



~~\$A~~

$A + B(2p-1)$