this time: Delta method, confidence intervals

next time: Markov chains

quiz 9 due Fri night; THT? due Sun night

AMS 131 28 Aug 19

①

social office hour tomorrow 5p→8p Lúpulo 233 Cathcart St., Santa Cruz

(9.43)

Neyman (–1981)

repeated sampling data set

population

sample

(X_i cont.)

$N = ?$ big

(actual) (like) IID

$X_1 \; Y_1$

$\vdots$

$X_n \; Y_n$

n

mean $\mu = ?$

SD $\sigma = ?$

pop. PDF

Gosset

hyp. IID

mean $\bar{X}_n$ $(\bar{y}_n)$ known

SD S_n

n

mean $\bar{X}_n$

all possible $\bar{X}_n$

$M \to \infty$

(WLLN)

long run mean $E(\bar{X}_n) = \mu$

estimated (WLLN)

long run SD $\sqrt{\hat{V}(\bar{X}_n)} = \frac{\sigma}{\sqrt{n}} \doteq \frac{S_n}{\sqrt{n}}$

long run PDF ⊛⊛

this is statistical inference: we want to draw inferential conclusions about μ (Based on data, μ is around $\underline{\bar{Y}_n\ (\bar{y}_n)}$ ①, give or take $\underline{\frac{\cancel{\sigma}\, s_n}{\sqrt{n}}}$ ②, and I'm highly (95%) $100(1-\alpha)\%$ confident that μ is between

$$\boxed{\underline{\bar{Y}_n - \cancel{\sigma}\frac{\sigma}{\sqrt{n}}}\ ③ \text{ and } \underline{\bar{Y}_n + \cancel{\sigma}\frac{\sigma}{\sqrt{n}}}\ ④}$$

$t^{-1}_{n-1}\left(1-\frac{\alpha}{2}\right)$

confidence interval (CI)

How probable is it that $\bar{Y}_n$ will differ from μ by more than _______?

$$E(\bar{Y}_n) = \mu \checkmark$$

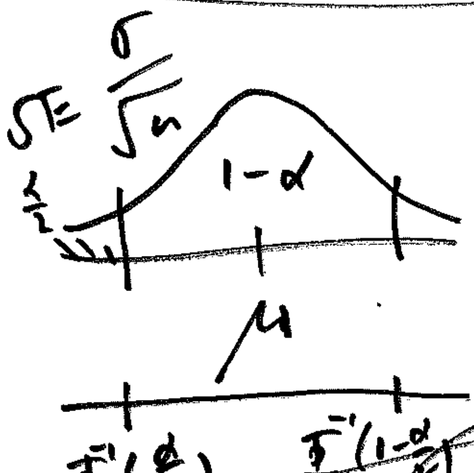
(*)

$$V(\bar{Y}_n) = \frac{\sigma^2}{n} \checkmark \qquad SD(\bar{Y}_n) = \sqrt{V(\bar{Y}_n)} = \frac{\sigma}{\sqrt{n}}$$

def if $\hat{\theta}_n$ estimates θ, $SD(\hat{\theta}_n) \doteq$ standard error (SE) of $\hat{\theta}_n$

$$SE(\bar{Y}_n) = SD(\bar{Y}_n) = \frac{\sigma}{\sqrt{n}}$$

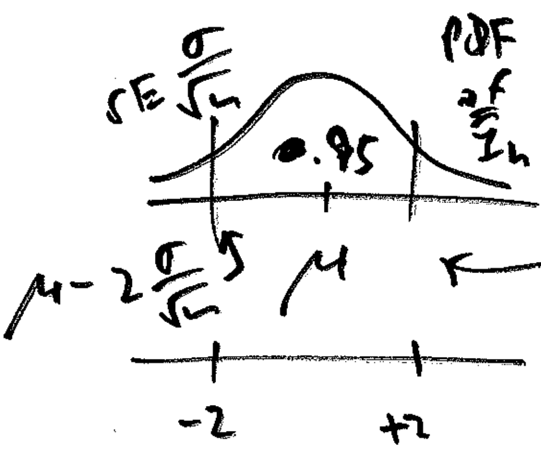
pick $0 < \alpha$, small



PDF of $\bar{Y}_n$, n large enough for CLT to apply

step 1: pretend σ known

ex. $\alpha = .05$



frequentist

$$P_F\left(\mu - 2\frac{\sigma}{\sqrt{n}} < \boxed{\bar{Y}_n} < \mu + 2\frac{\sigma}{\sqrt{n}}\right) \doteq 95\%$$

Neyman's confidence trick

random variable

$$P_F\left(\bar{Y}_n - 2\frac{\sigma}{\sqrt{n}} < \mu < \bar{Y}_n + 2\frac{\sigma}{\sqrt{n}}\right) \doteq 95\%$$

$\therefore$ let's define $\left(\bar{Y}_n \pm 2 \frac{\sigma}{\sqrt{n}} \right)$ to be a 95% confidence interval for μ

Laplace 1800

easy to conclude that $P_F (8.4 < \mu < 11.6) \doteq 95\%$

but this is wrong: ***

$\bar{Y}_n = 10$

$n = 25$

$\sigma = 4$

$$\bar{Y}_n \pm 2 \frac{\sigma}{\sqrt{n}} = 10 \pm 2 \frac{4}{\sqrt{25}}$$

$$= (8.4 , 11.6)$$

$\leftarrow$ 95% CI for μ $\rightarrow$

(8.4 | 10) 11.6

*** μ is a fixed unknown constant; $\therefore$ no frequentist probability statements can be made about it: $P_F (8.4 < \mu < 11.6) =$ undefined

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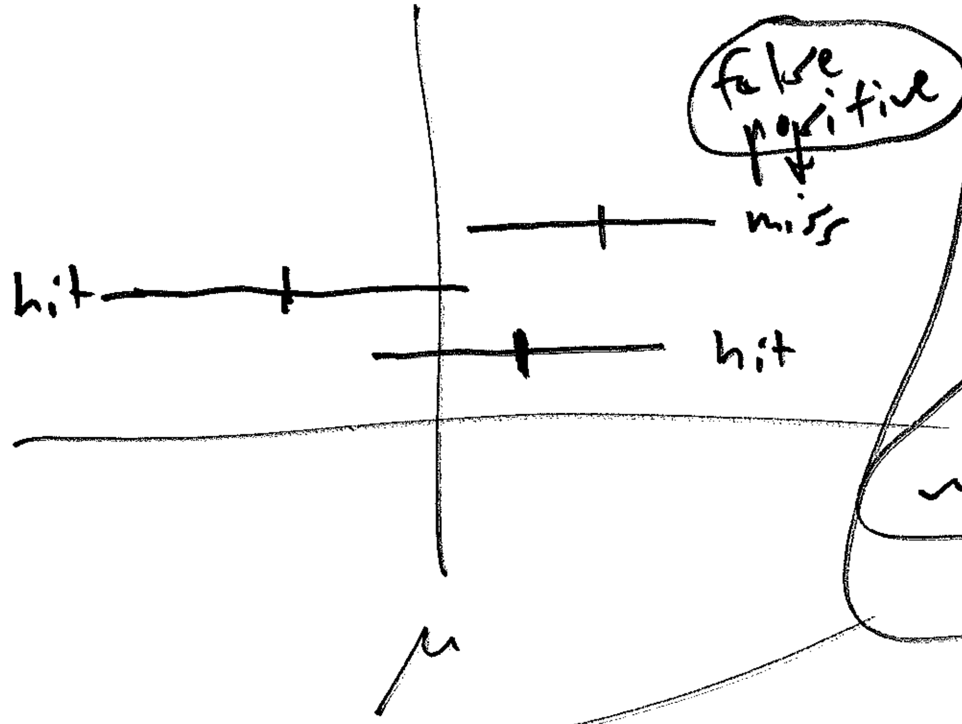


95% confidence ↔ 95% hit rate across many CIs

we will not know if *our* 95% CI is a hit or a miss; our confidence is in the *process* of building the CI, not the outcome (the CI itself)

$100(1-\alpha)\%$ CI: $\bar{X}_n \pm \Phi^{-1}(1-\frac{\alpha}{2})\frac{\sigma}{\sqrt{n}}$

~~all~~ $100(1-\alpha)\%$ CI have $100\alpha\%$ false-positive rate

Fisher's convention: 95% ~~CI~~

95% CI : too low for good, replicable science

(.05) false (+) rate

recent suggestion :

.05 / 10 → .005

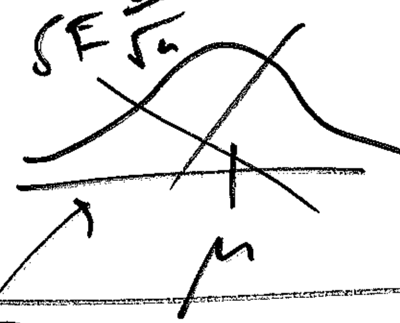
99.5% CI

99.9%

99.7% (±3)

step 2 : σ unknown

SE $\frac{\cancel{\sigma}\, s_n}{\sqrt{n}}$



PDF of $\bar{X}_n$ & (CLT)

$\bar{X}_n \pm \boxed{\cancel{\Phi^{-1}\left(1-\frac{\alpha}{2}\right)}}\ \frac{\cancel{\sigma}\, s_n}{\sqrt{n}}$

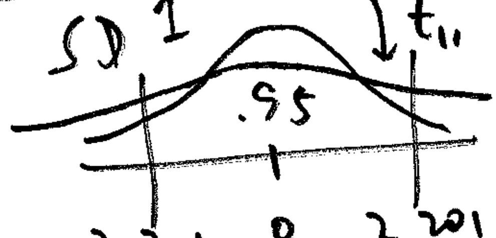
$t^{-1}_{n-1}\left(1-\frac{\alpha}{2}\right)$

n large → $\Phi^{-1}\left(1-\frac{\alpha}{2}\right)$ approx. ok

n small

William Gosset (1908)

t_{n-1} curve



PDF of $\frac{\bar{X}_n - \mu}{s_n / \sqrt{n}}$

~~(CLT)~~, n small, PDF of $\bar{X}_n$ approx. N

$t_{(n-1)}$ ← degrees of freedom

as n ↑

$t_n \to \Phi$

(11.03)