Discussion Section 9

roulette single-#

AMS 131
27 Aug 19

①

pop: possible your net gain on single play

$$\left[\begin{array}{c} -\$1 \\ \vdots \\ -1 \\ +\$35 \end{array}\right] \quad 37 \text{ (36 of } -1\text{, 1 of } +35\text{)}$$

IID → $\begin{array}{c} s_1 \\ \vdots \\ s_n \end{array}$ $n = 1{,}000$

sample: the observed net gains

sum $S = ?$ ex. $+\$8$

IID → $n = 1{,}000$

sum $S' = ?$ (ex. $-\$28$)

repeated-sampling dataset

$$\left[\begin{array}{c} +88 \\ -28 \\ \vdots \end{array}\right] \quad M \to \infty$$

long-run mean: $E(S) = n\mu = -\$52$

long-run SD: $\sqrt{V(S)}$

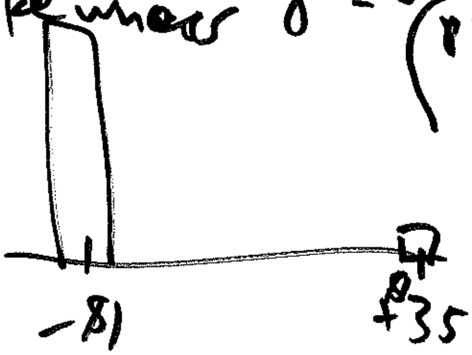
mean $\mu \doteq -\$0.05$

SD $\sigma = \$5.76$

skewness $\gamma = 5.92$

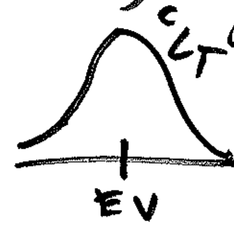
pop PMF

kurtosis $\kappa = 33.0$



if n is big enough CLT

long-run PMF of S



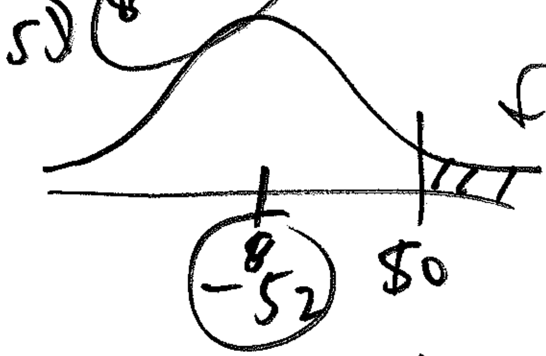
$$P(\text{coming out ahead}) = P(S > \$0) \doteq \frac{\#(\text{simulated sums} > \$0)}{M}$$

$$SD(S) = \sqrt{V(S)} = \sigma\sqrt{n}$$

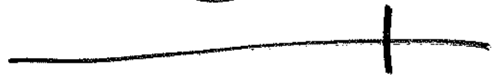
$$= \$182$$

sd $\$182$

PMF of S



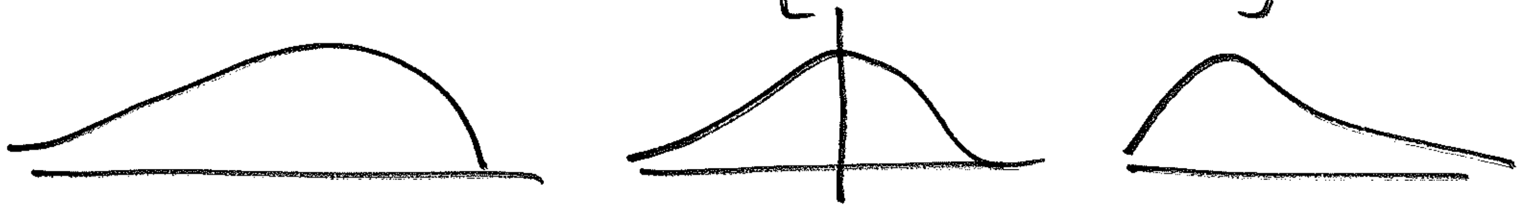
$0.3864 \doteq 39\%$



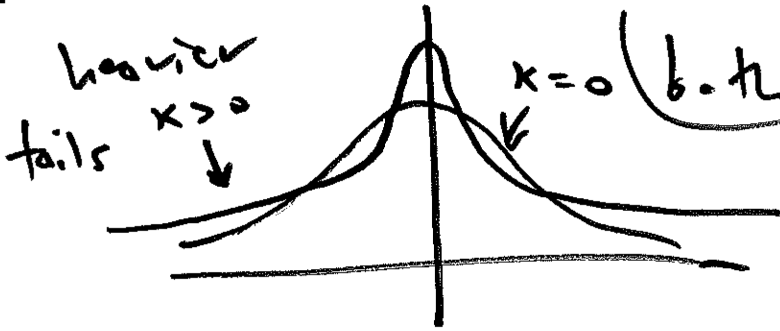
$+0.2888 \leftarrow \dfrac{\$0 - (-\$52)}{\$182}$

$$\text{skewness}(Y_i) = \text{population skewness}$$

$$= \gamma = E\left[\left(\frac{Y_i - \mu}{\sigma}\right)^3\right]$$



$\gamma < 0$ | skewness $\gamma \doteq 0$ (symmetric) | $\gamma > 0$



both have skewness 0

4th moment:

kurtosis = κ

tail weight relative to normal curve

$$\kappa = E\left[\left(\frac{X_i - \mu}{\sigma}\right)^4\right] - 3$$

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \; ; \quad S_n = \sum_{i=1}^{n} X_i$$

$$S_n = n\bar{X}_n$$

$$\text{skewness}(\bar{X}_n) = \text{skewness}(S_n) = \frac{\text{skewness}(X_1)}{\sqrt{n}} = \frac{\gamma}{\sqrt{n}}$$

pop. skew

set $\frac{\gamma}{\sqrt{n}} = c_{skew} \doteq 0$

$$\text{kurtosis}(\bar{X}_n) = \text{kurtosis}(S_n) = \frac{\text{kurtosis}(X_1)}{n} = \frac{\kappa}{n}$$

$\leftarrow$ pop. kurt.

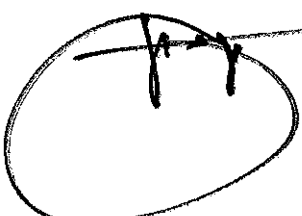
set $\frac{\kappa}{n} = c_{kurt} \doteq 0$

$$n_{skew} = \left(\frac{\gamma}{c_{skew}}\right)^2 \qquad n_{kurt} = \frac{\kappa}{c_{kurt}}$$

ex. $c_{skew} = 0.1$ or 0.01 ; $c_{kurt} = 0.1$ or 0.01

recommended minimum n for CLT to give good normal approx $= \max(n_{skew}, n_{kurt})$

red-or-black



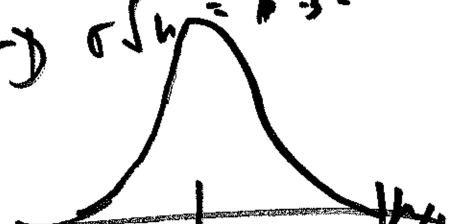
-1
⋮ 20
-1

+1
⋮ 18
+1

mean $\mu = -\$0.05$

-1 +1

SD $\sigma = \$1.00$

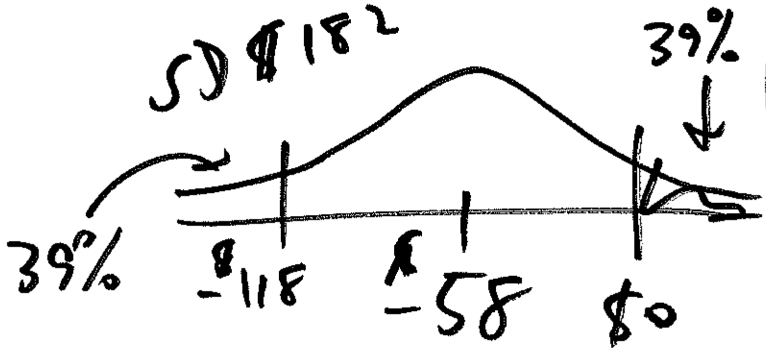


SD $\sigma\sqrt{n} = \$32$

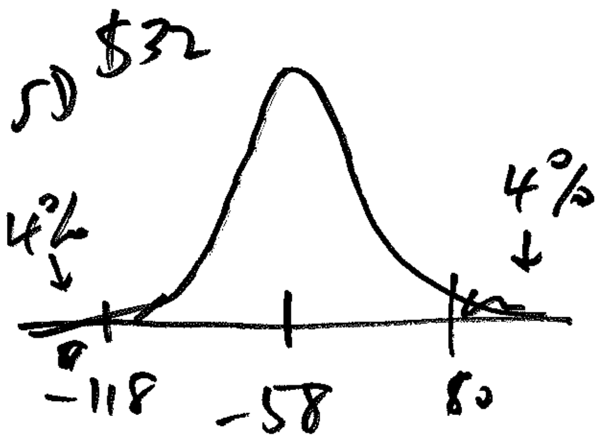
$-\$52.63$ EV

\$0

PMF of S (red 2 black) ($n = 1{,}000$)



approx. PMF of $, single #, n = 1,000



approx PMF of $, red-or-black, n = 1,000