this time: Poisson, Negative Binomial, Geometric, Normal, Gamma,

next time: Beta, Multinomial, Bivariate Normal

THT 2 due TONIGHT; Quiz 7 due tomorrow night.

quiz 8 due next Tue

today: extra notes p. 243+

(9.44) PDF $X$

SD $\sigma$

$\mu$

deMoivre (1718)

$$f_X(x \mid \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[ -\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2 \right]$$

converting to standard units (SU)

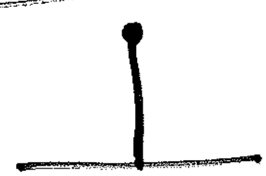
"Normal" 1890 (Karl Pearson)

"Gaussian" (1790)

$-\infty < \mu < +\infty$

$\sigma \geq 0$

$\sigma = 0$



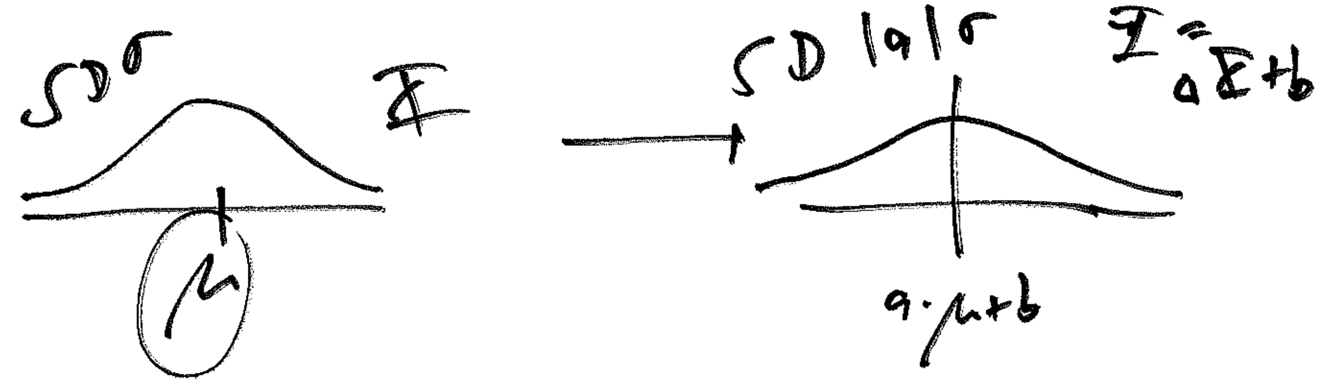
$\mu$

let's use $\sigma > 0$

$$(X \mid \mu, \sigma^2) \sim N(\mu, \sigma^2),$$

$$Y = aX + b, \quad \begin{pmatrix} a \neq 0 \\ b \end{pmatrix} \text{ fixed constants}$$

$\rightarrow$

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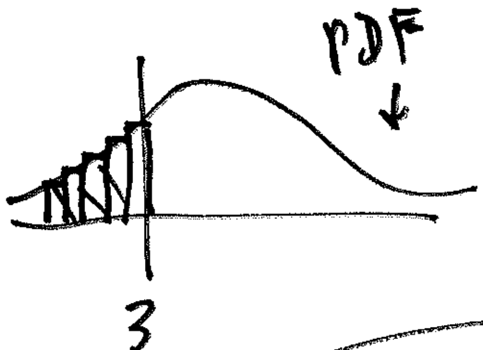


$$\boxed{X \sim N(\mu, \sigma^2)}$$

$$P(X \leq 3) = ?$$

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CDF of Normal$(\mu, \sigma^2)$ has no closed form

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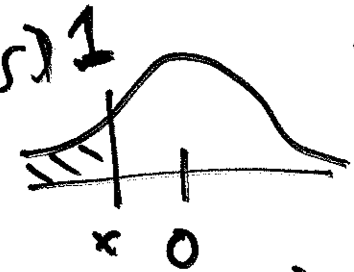
$$\boxed{\Gamma(x) = \int \text{——}}$$




standard Normal dist.

capital phi! $\rightarrow$

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{t^2}{2}\right] dt$$

$x = \begin{cases} 1 & \text{if } y \le 3.5 \\ 0 & \text{else} \end{cases}$

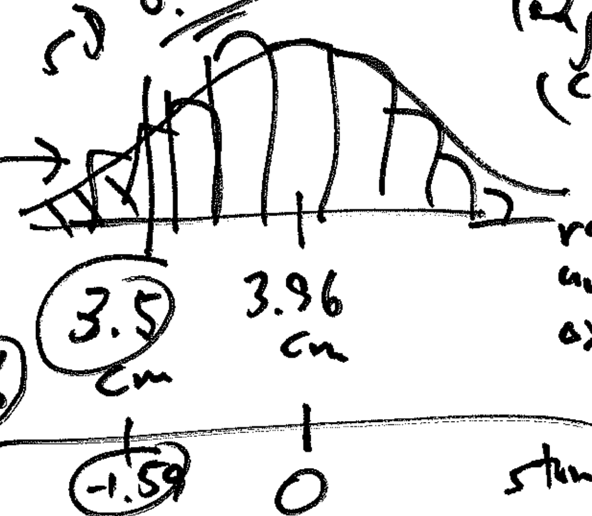
| $y$ | $x$ |
|---|---|
| $y_1 = 4.1$ | 0 |
| $y_2 = 3.3$ | 1 |
| $\vdots$ | 1 |
| | 1 |
| $y_n = 4.7$ | 0 |

$n = 103$

mean $\bar{y} = 3.96\text{cm}$

SD $s = 0.29\text{cm}$

mean 7.8%

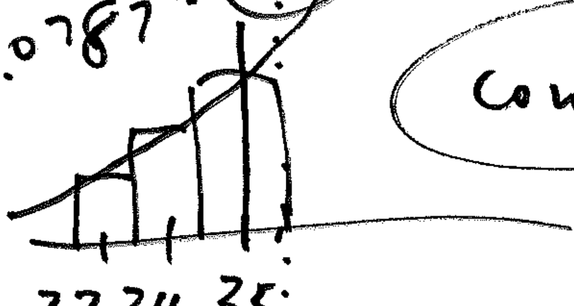



$.0563 \doteq 5.6\%$

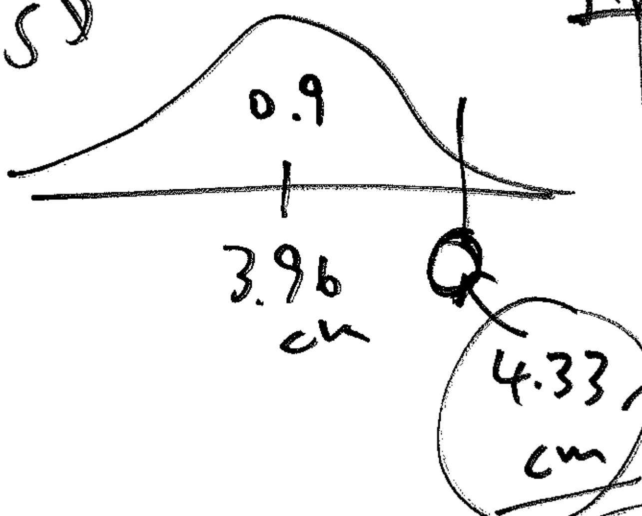
$$z = \frac{y - \bar{y}}{s}$$

$$= \frac{3.5\cancel{\text{cm}} - 3.96\cancel{\text{cm}}}{0.29\cancel{\text{cm}}}$$

$.0787 \doteq 7.9\%$




continuity correction

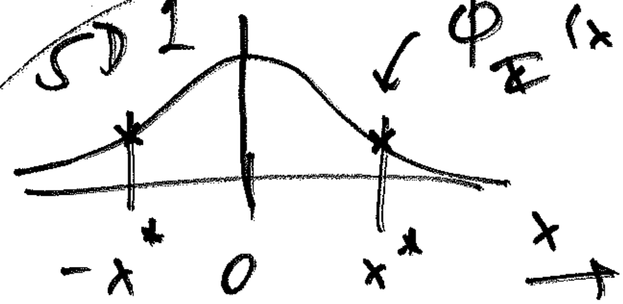



**Q:** what is the 90th percentile of the wing length dist.?

inverse CDF

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$$

$$\Phi^{-1}(p) = ?$$

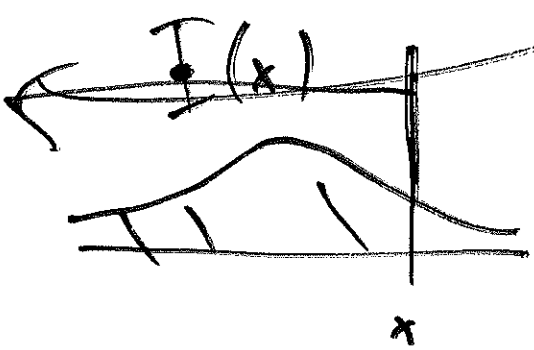


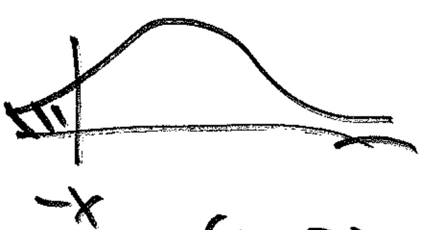

for all $x \geq 0$

$$\phi_Z(x) = \phi_Z(-x)$$

$$\Phi(x) + \Phi(-x) = 1$$

for all $x$




(10.50)

pop

sample

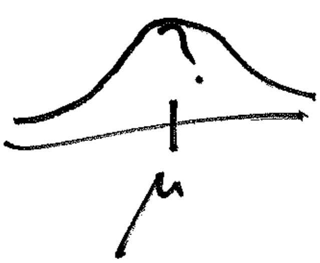
$X_1, \ldots, X_n$ IID $\quad x_1, \ldots, x_n \quad n$

mean $\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i \quad (\bar{x})$

mean $\mu = ?$

SD $s =$

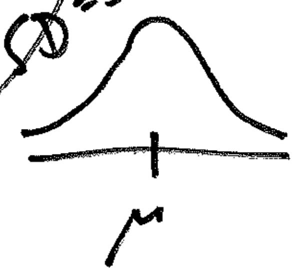
SD $\sigma = ? < \infty$



pop. PDF

sample hist.

SD = SE = ?



PDF of $\bar{X}_n$

intuition: $\bar{X}_n$ is a good estimate of $\mu$

$$E(\bar{X}_n) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n} E\left(\underline{\sum_{i=1}^{n} X_i}\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(X_i) \overset{\text{ID}}{=} \frac{1}{n}\sum_{i=1}^{n} \mu = \mu \checkmark$$

$E(\bar{X}_n) = \mu$ $\therefore$ $\bar{X}_n$ is <u>unbiased</u> for $\mu$

$$SD(\bar{X}_n) \doteq SE(\bar{X}_n) = \frac{c}{f(n)} = ?$$

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$$SD\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n} SD\left(\sum_{i=1}^{n} X_i\right) = ?$$

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$$V(\bar{X}_n) = V\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$$

$$= \frac{1}{n^2} V\left(\sum_{i=1}^{n} X_i\right)$$

$$\overset{\text{I}}{=} \frac{1}{n^2}\sum_{i=1}^{n} V(X_i)$$

$$\overset{\text{II}}{=} \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2 = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

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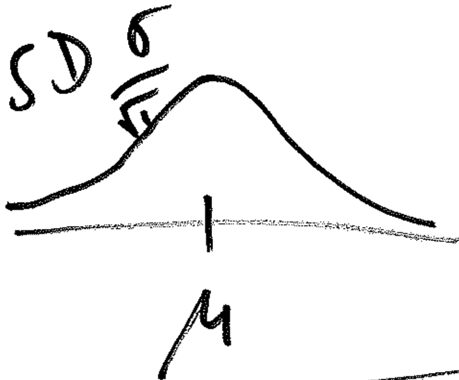
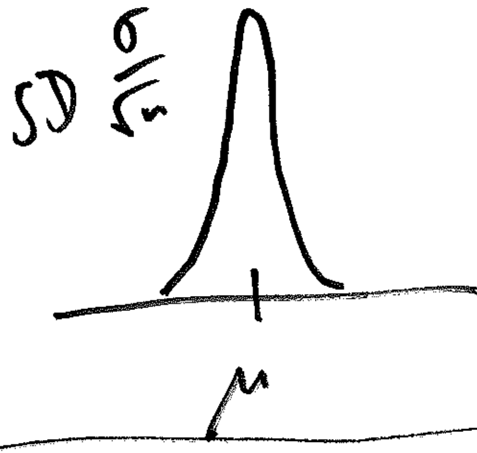
$$SD(\bar{X}_n) = SE(\bar{X}_n) = \sqrt{V(\bar{X}_n)}$$

$$= \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}} \leftarrow \left(\text{square root law}\right)$$

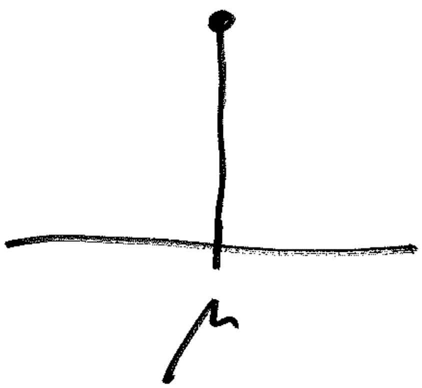



PDF of $\bar{X}_1$ (pop. PDF)




PDF of $\bar{X}_n$ $(n > 1)$

assumption: $\sigma < \infty$




$n \to \infty$