Discussion Section 8

AMS 131
22 Aug 19

(1)

haphazard individuals with respect to weight ⟺ like at random

population: all adult Pimlico tube users in rush hour (1995)

sample: the observed people

repeated-sampling data set

weight

$N = ?$ (big)

like at random (like IID)

weight

I_1 y_1

⋮ ⋮

I_n y_n

$n = 192$

sum $S = ?$ (ex. 30,800)

30,800
32,045
⋮

$M \to \infty$

mean $\mu = 158$ lb.
SD $\sigma = 33$ lb.

IID

$n = 192$

sum $S = ?$ (ex. 32,045)

real world

population PDF

$f_{I_i}(y_i)$

long run mean: $E(S) = n\mu$

long run SD: $SD(S) = \sigma\sqrt{n}$

long-run PDF of S

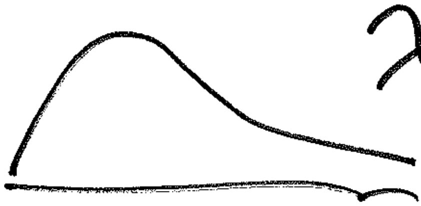
The total weight of people on a fully-loaded escalator run $\boxed{\text{is like}}$ the sum S of n IID draws from this population

math world

real world

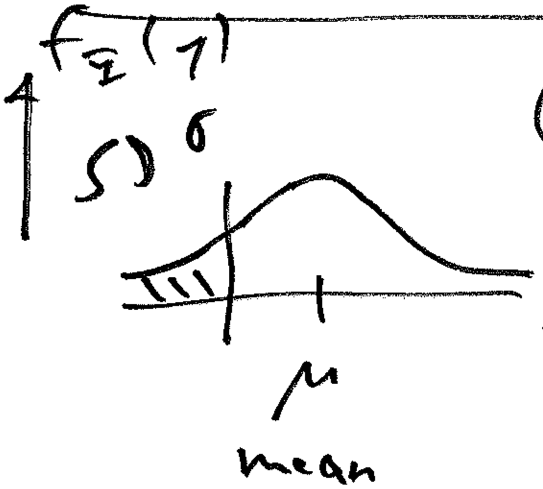
$P(\text{escalator breaks down}) = P(S > 31{,}400) = ?$

$Y_i \overset{IID}{\sim} f_{Y_i}(y)$, $\quad S = \sum_{i=1}^{n} Y_i$



χ^2 dist

$$\hat{P}(S > 31{,}400) = \frac{\#\, S \text{ values} > 31{,}400}{M} \doteq .010$$



(deMoivre (1701))

$-\infty < \mu < +\infty \qquad \sigma > 0$

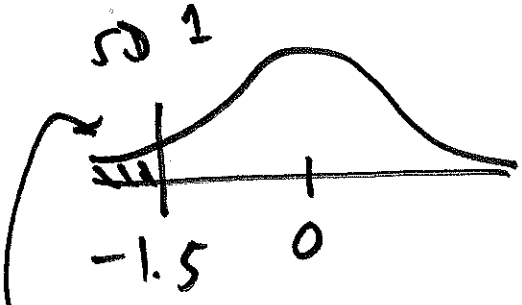
$$f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(y_i-\mu)^2}{2\sigma^2}\right]$$

$(Y \mid \mu, \sigma^2) \sim \underline{\text{Normal}}(\mu, \sigma^2)$

$\sim N(\mu, \sigma^2)$ variance

$g(y) = e^{-y^2}$

CDF of $N(\mu, \sigma^2)$ has no closed form: need numerical integration to evaluate $F_Y(y)$



standard Normal PDF $= N(0,1)$

$.0668 \doteq 6.7\%$

$$S = \sum_{i=1}^{n} T_i$$

$$E(T_i) = \mu$$

$$V(T_i) = \sigma^2$$

$$E(S) = E\left(\sum_{i=1}^{n} T_i\right)$$

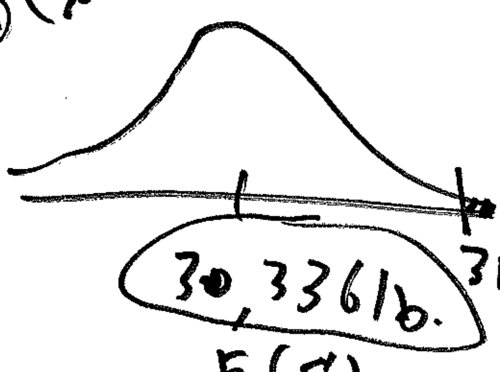
$$= \sum_{i=1}^{n} E(T_i) = \sum_{i=1}^{n} \mu = \boxed{n\mu = E(S)}$$

$= 192(158 \text{ lb.})$

$= 30{,}336 \text{ lb.}$

$SD(S) = 457 \text{ lb.}$

PDF of S



$SD(S) =$

$SD\left(\sum_{i=1}^{n} T_i\right)$

$= ?$

$$V\left(\sum_{i=1}^{n} T_i\right) \overset{\text{I}}{=} \sum_{i=1}^{n} V(T_i) = \sum_{i=1}^{n} \sigma^2 = n\sigma^2$$

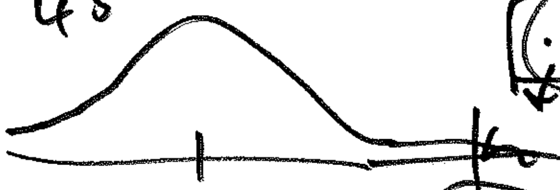
$$SD(S') = \sqrt{V(S')} = \sqrt{n\sigma^2}$$

$$= \sigma\sqrt{n} \to (33\text{ lb.})\sqrt{192} \doteq 457\text{ lb.}$$

.990

SD 457

.010



30,336 lb. 31,400 lb.

raw units (y) (lb.)

$$y \to \frac{y-\mu}{\sigma} = z_y$$

$$y = \mu + \sigma z_y$$

standard units (z)

0 +2.33

$$= \frac{31400 - 30336}{457}$$

SD 457

.0001

30,336

$y =$ 32,036 lb.

+3.719

failure rate	critical weight
.01	31,400
.0001	32,036