Discussion Section 7

Extra notes pp. 241 → 282 →

AMS 131, 20 Aug 19

①

$100/220 \doteq 45.5\%$

if ethnicity-neutral (*) → T_1 (pretend true)

eligible pop.: all people in H.C. catchment area in 1974

sample: the observed people

M.-A.? (1 = yes, 0 = no)

$N = ?$ elig. : 1s, 0s

like IID → M.-A.?: $Y_1, \dots, Y_n$: 1s, 0s, $n = \underline{220}$

see if data is plausible

sum $S = 100 = X$

mean $\bar{Y} = 0.455$

mean $\theta = 0.791$

IF (*):

$$(Y_i \mid T_1) \overset{IID}{\sim} \text{Bernoulli}(\theta) \text{ and}$$

$$(X \mid T_1) = \sum_{i=1}^{n} Y_i \sim \text{Binomial}(n, p), \quad n = 220$$

$$E(X \mid T_1) = np = (220)(0.791) \doteq 174.0$$

$$SD(X \mid T_1) = \sqrt{np(1-p)} = \sqrt{220(.791)(.209)} = 6.0$$

p-value = P(if T_1 true, of getting data as extreme as, or more extreme than, what we got) = $8 \cdot 10^{-28}$

① pretend T_1 true | ② if T_1 true, data unbelievably unlikely

③ ∴ T_1 probably false | probability version of proof by contradiction

② Compute p-value ($8 \cdot 10^{-28}$)

if p is small T_1 looks bad

how small?

Convention: $p \leq 0.05$, "reject T_1" | say T_1 true when actually it's false

say T_1 false when true

false rejection of T_1 ↔ false positive ③

0.05 ?

① Fisher told us to use it.

② 5 fingers

$\frac{.95}{.05} = 19$ to 1

replicability crisis

better choice → greater replicability

$p \leq 0.005$

T_1 often your theory: ↔ something new in world

try null often for size

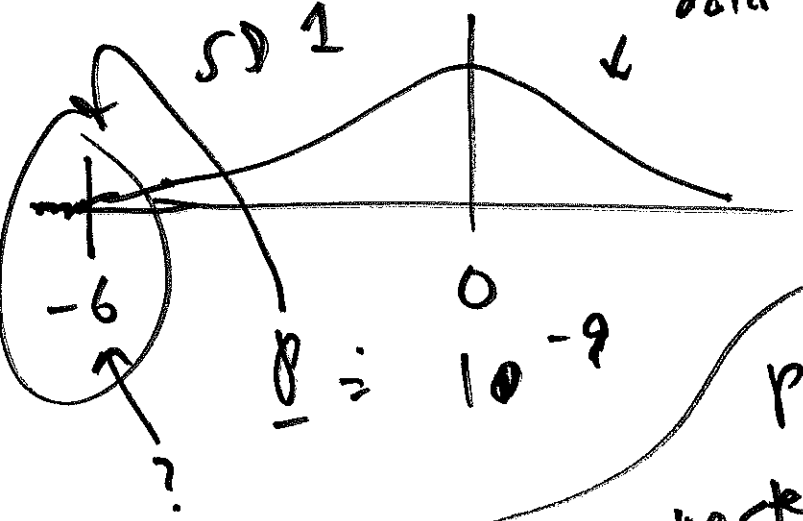
null hypothesis: status quo (my new theory wrong)

alternative hypothesis: my new theory right

if $p \leq 0.05$, reject null: claim my theory correct

powerful incentive to reject null

p-hacking



$\theta \doteq 10^{-9}$

PDF $\left(\begin{array}{c}\text{prior info.}\\ \text{about } \theta\end{array}\right)$ $\left(\begin{array}{c}\text{data}\\ \text{info.}\end{array}\right)$

$$p(\theta|s) = \frac{p(\theta)\,p(s|\theta)}{p(s)}$$

$p(\theta|s)$: posterior info. about θ given s

$p(s)$: normalizing constant

$(c>0)$

$$p(\theta|s) = c \cdot p(\theta) \cdot \ell(\theta|s)$$

$p(\theta)$: info. about θ external to s

$\ell(\theta|s)$: info. about θ internal to s

Laplace / Fisher

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$$\ell(\theta|s) = c \cdot p(s|\theta)$$

$\ell(\theta|s)$: likelihood function

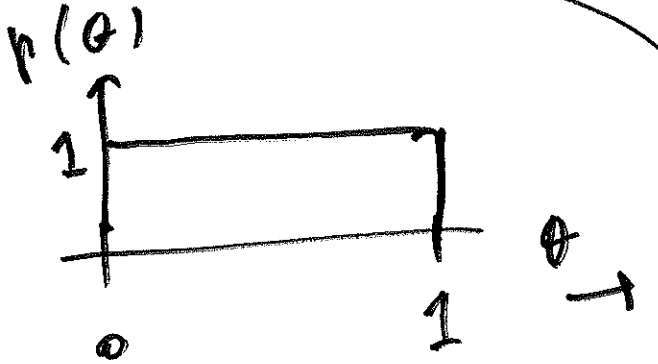
$(c>0)$

$$f_{s|\theta}(s|\theta) = p(s|\theta)$$

$$\ell(\theta|s) = c\binom{n}{s}\theta^{s}(1-\theta)^{n-s}$$

$$p(\theta|s) = c\, p(\theta)\, \theta^{s}(1-\theta)^{n-s}$$ (5)

pdf



$p(\theta) = 1 \cdot I(0<\theta<1)$

neutral prior

with this prior,

$p(\theta|s) = c\, l(\theta|s)$

Bayes (17 Oct)

$$p(\theta|s) = c\, \boxed{p(\theta)}\, \theta^{s}(1-\theta)^{n-s}$$

$$c\,\theta^{(\alpha+s)-1}(1-\theta)^{(\beta+n-s)-1} \quad * \Rightarrow \theta^{\alpha-1}(1-\theta)^{\beta-1}\,\theta^{s}(1-\theta)^{n-s}$$

$$h(\alpha,\beta) = \int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1}\, d\theta$$