Discussion Section 6

AMS131 15 Aug 19 1

$$f_X(x) = \begin{cases} \dfrac{(x+10)^2}{9000} & \text{for } -10 \le x \le +20 \\ 0 & \text{else} \end{cases}$$

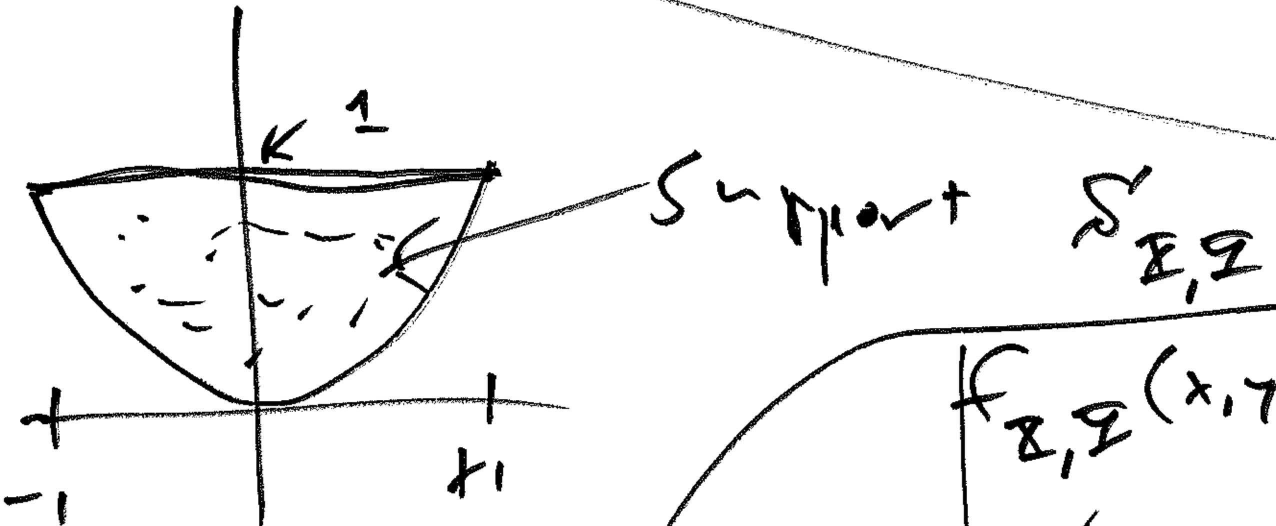
$$F_X(x) = \begin{cases} 0 & \text{for } x \le -10 \\ \dfrac{(x_p+10)^3}{27000} = p & -10 \le x \le +20 \\ 1 & x \ge +20 \end{cases}$$

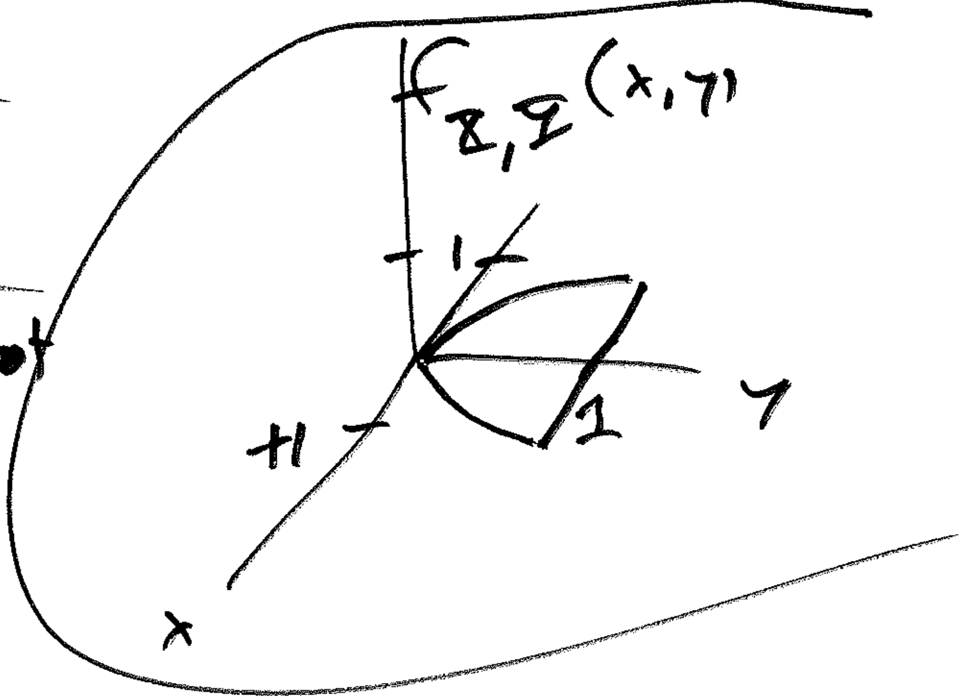
$X \sim F_X$ $\qquad F_X(X) \sim U(0,1)$

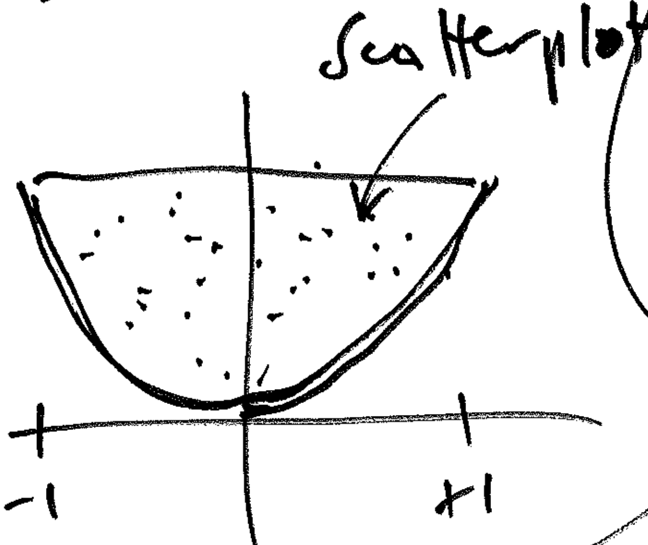
$X \sim F_X^{-1}\left[U(0,1)\right]$

$$\frac{(x_p+10)^3}{27000} = p \qquad x_p = F_X^{-1}(p) = 30\,p^{\frac{1}{3}} - 10$$

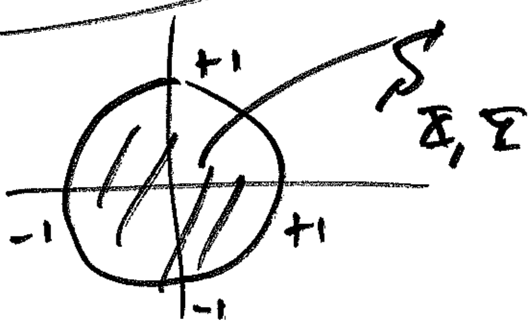
$$f_{X,Y}(x,y) = \begin{cases} \dfrac{21x^2y}{4} & 0 \le x^2 \le y \le 1 \\ 0 & \text{else} \end{cases}$$ for ②





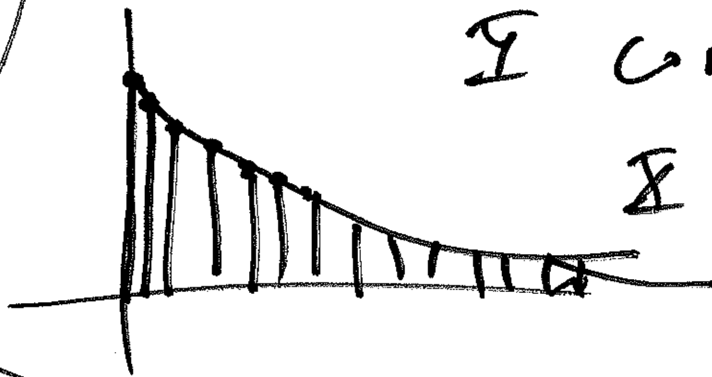


$$f_{X,Y}(x,y) = \begin{cases} \frac{24}{\pi} x^2 y^2 & 0 \le x^2 + y^2 \le 1 \\ 0 & \text{else} \end{cases}$$



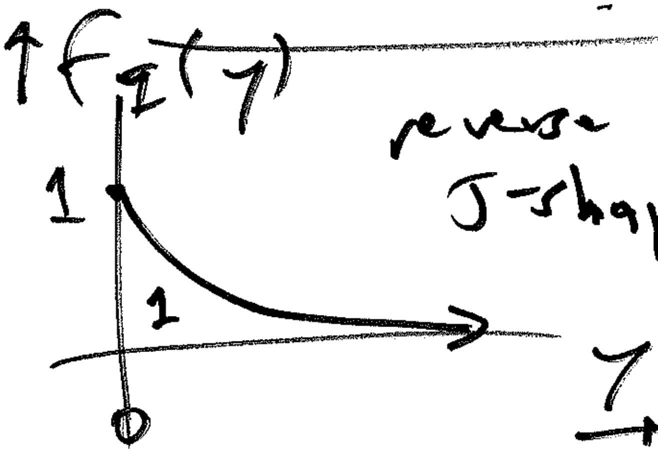
DS p. 152
#12

Y continuous
X discrete



$$f_Y(y) = \begin{cases} e^{-y} = \exp(-y) & \text{for } y > 0 \\ 0 & \text{else} \end{cases}$$

PDF of Y



reverse J-shape

$$\int_0^\infty e^{-y}\,dy = 1 \checkmark$$

$Y \sim$ Exponential(1)

$$f_Y(y \mid \lambda) = \begin{cases} \lambda e^{-\lambda y} & \text{for } y > 0 \\ 0 & \text{else} \end{cases}$$

$P(B_i) \to y$

~~$P(X = x \mid Y = y)$~~

$$f_{X|Y}(x \mid y) = \begin{cases} \dfrac{(2y)^x}{x!} e^{-2y} & \text{for } x = 0, 1, \ldots \\ 0 & \text{else} \end{cases}$$

$P(A \mid B_i)$

$$\sum_{x=0}^{\infty} \frac{(2y)^x}{x!} e^{-2y} = 1 \quad (\text{for fixed } y > 0)$$

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$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$= \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

$$f_X(x) = ?$$

$P(A)$

$$P(A) = \sum_{i=1}^{n} P(B_i) P(A \mid B_i) \quad \text{LTP}$$

y x y

fix $x = 0, 1, \dots$

$$f_X(x) = \int_0^{\infty} f_Y(y) f_{X \mid Y}(x \mid y)\, dy$$

$$= \int_0^{\infty} e^{-y} \frac{(2y)^x}{x!} e^{-2y}\, dy$$

$$= \frac{2^x \cdot 3^{-(x+1)} \Gamma(x+1)}{x!}$$

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but since $x = 0, 1, \ldots$

$\Gamma(x+1) = x!$

$2^x \cdot 3^{-(x+1)}$

$= \left(\frac{2}{3}\right)^x \frac{1}{3}$

$$f_X(x) = \begin{cases} \left(\frac{2}{3}\right)^x \cdot \frac{1}{3} & \text{for } x = 0, 1, \ldots \\ 0 & \text{else} \end{cases}$$

$$f_{Y|X}(y \mid x) = ?$$

we know

$f_Y(y)$

$f_{X|Y}(y)$

$f_X(x)$

$$f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

? use Bayes's Thm

$$f_{Y|X}(y \mid x) = \frac{f_Y(y)\, f_{X|Y}(x \mid y)}{f_X(x)}$$

for fixed $x = 0, 1, \dots$

$$f_{Y|X}(y|x) = \begin{cases} \dfrac{\cancel{e^{-y}}\,(2y)^x\,\cancel{e^{-2y}}}{\cancel{x!}\cdot\left(\frac{2}{3}\right)^x \cdot \cancel{\frac{1}{3}}} & \text{for } y > 0 \\[2ex] 0 & \text{else} \end{cases}$$

$$\frac{3 \cdot e^{-3y}\, y^x\, 3^x}{x!}$$

$$\frac{(2y)^x\, 3^x}{2^x}$$

$$f_{Y|X}(y|x=0) = 3e^{-3y}$$

$$f_{Y|X}(y|x=1) = 3e^{-3y}\, y\, 3$$

ratio $> 1 \longleftrightarrow y > \frac{1}{3}$

$$\frac{f_{Y|X}(y|x=1)}{f_{Y|X}(y|x=0)} = \frac{3\cdot\cancel{3}\,\cancel{e^{-3y}}\,y}{\cancel{3}\,\cancel{e^{-3y}}} = 3y$$