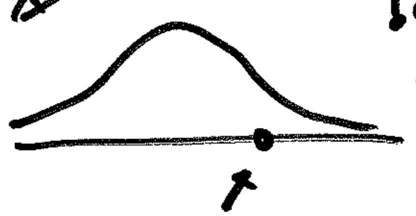
AMS 131
12 Aug 19

this time: joint,
2 RV:
next time: marginal,
conditional distributions

quiz 4 due at canvas by 11:59 pm tomorrow (1) night (wed); take home

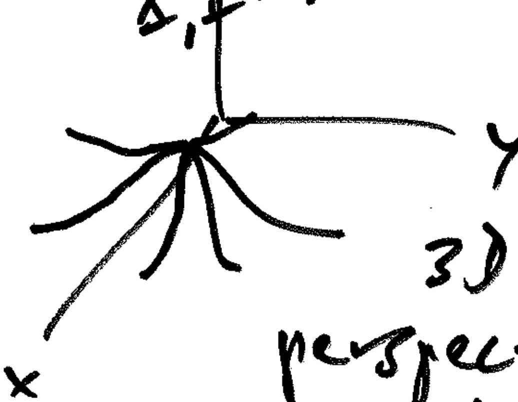
test 1 due by 11:59 pm 2 nights from now

$f_X(x)$ univariate "bell curve"

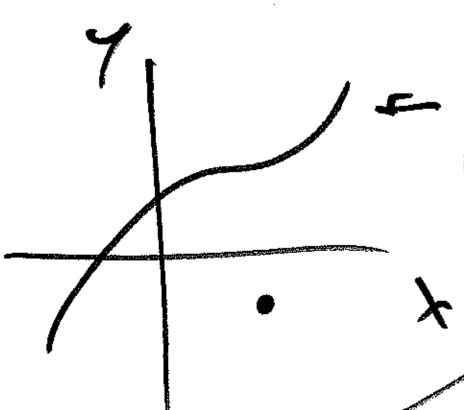


bivariate "bell curve"

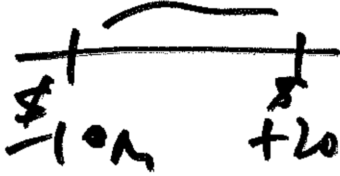
$f_{X,Y}(x,y)$



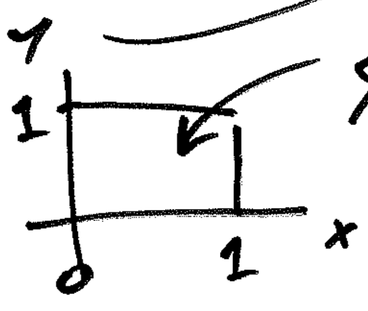
3D perspective plot



S support



$f_X(x) = \begin{cases} \sim & \text{for } x \in S \\ 0 & \text{else} \end{cases}$

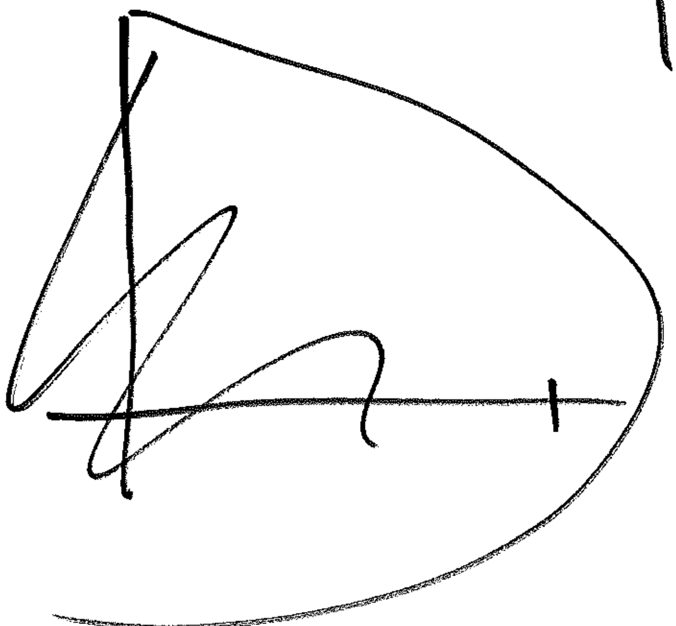


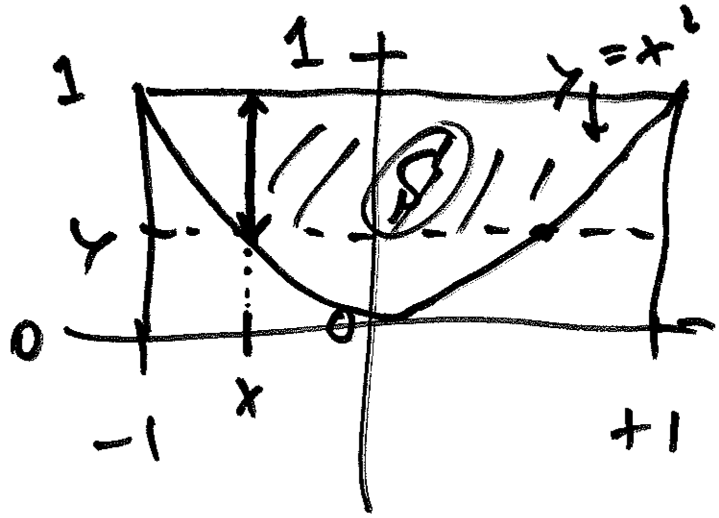
$S = \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1 \end{cases}$

$1 = \int_0^1 \int_0^1 f_{XY}(x,y)\,dx\,dy$

for ②

$$f_{X,Y}(x,y) = \begin{cases} \frac{21}{4} x^2 y & 0 \le x^2 \le y \le 1 \\ 0 & \text{else} \end{cases}$$





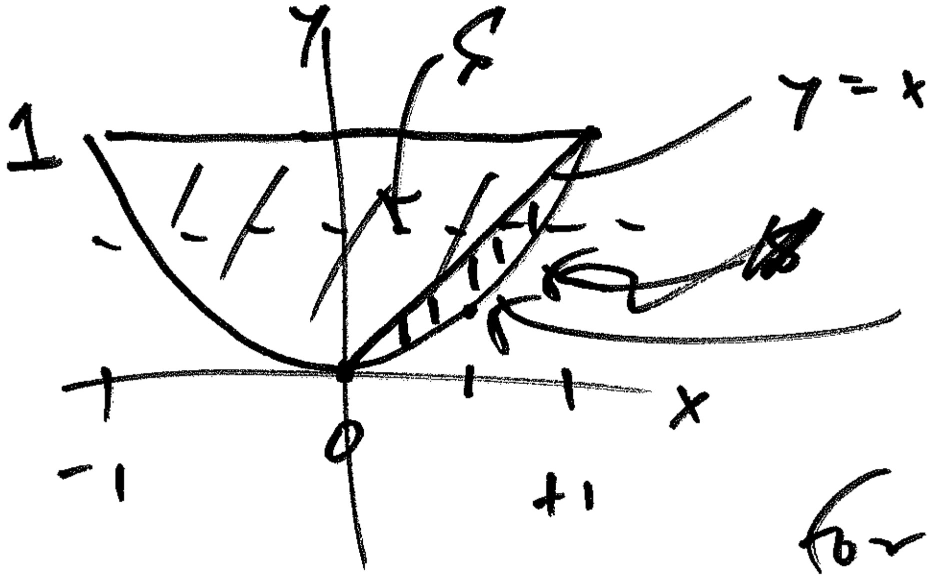
$$1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (c x^2 y)\,dx\,dy$$

S

$$= c \int_{-1}^{+1} \left[\int_{x^2}^{1} (x^2 y)\,dy\right] dx = \frac{4c}{21} = 1$$

$$c = \frac{21}{4}$$

$$1 = c \int_{0}^{1} \left[\int_{-\sqrt{y}}^{+\sqrt{y}} (x^2 y)\,dx\right] dy$$



interesting region for $P(X \ge Y)$

$$P(X \ge Y) = \int_0^1 \left[\int_{x^2}^{x} \frac{21}{4} x^2 y \, dy \right] dx = \frac{3}{20}$$

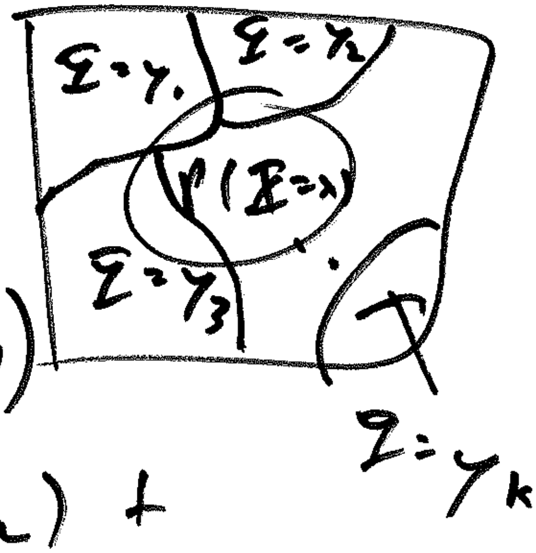
$$= \int_0^1 \left[\int_{y}^{\sqrt{y}} \frac{21}{4} x^2 y \, dx \right] dy = \frac{3}{20}$$

(4.53)

to get the marginal of X from the joint for (X, Y), sum (discrete) or integrate over y

$$f_{X,Y}(x,y) = P(X = x \text{ and } Y = y)$$

$$f_X(x) = P(X = x)$$



$$= P(X = x \text{ and } Y = y_1)$$

$$+ P(X = x \text{ and } Y = y_2) +$$

$$\cdots + P(X = x \text{ and } Y = y_k)$$

$$= \sum_{\text{all } y} P(X = x \text{ and } Y = y)$$

$$f_X(x) = \sum_{\text{all } y} f_{X,Y}(x,y)$$

(marginalizing over Y)

---

cont.

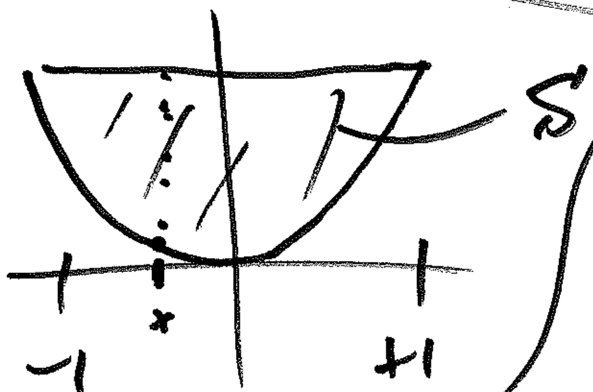
$$\underset{\sim}{X} = (X_1, X_2, \ldots, X_n)$$

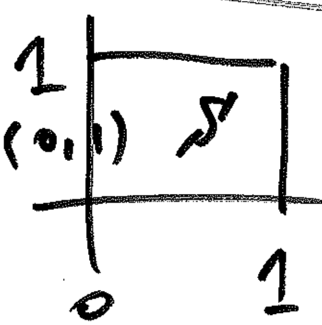
$$f_{X_1}(x_1) = \int_{x_2} \int_{x_3} \cdots \int_{x_n} f_{\underset{\sim}{X}}(x)\, dx_2 \cdots dx_n$$

$$f_X(x) = \int f_{XY}(x,y)\,dy$$

pick an x

$$= \int_{x^2}^{1} \frac{21}{4} x^2 y \, dy$$





$Y \sim U(0,1)$

$X \sim U(0,1)$

$$f_{XY}(x,y) = \begin{cases} 1 & \text{for } 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

why is independence important?

$\underset{\sim}{X} = (X_1, \dots, X_n)$ discrete

$f_{\underset{\sim}{X}}(\underset{\sim}{x}) = ?$

ow if not indep ow

$$f_{X_1,\dots,X_n}(x_1,\dots,x_n) = P(X_1 = x_1, \dots, X_n = x_n)$$

$$P(X_1 = 1)\, P(X_2 = x_2 \mid X_1 = 1)\, P(X_3 = x_3 \mid X_1 = x_1, X_2 = x_2) \cdots$$

Gender & MCP

$$X = \begin{cases} 1 & \text{if } F \\ 0 & M \end{cases}$$

$$Y = \begin{cases} 1 & \text{if } Y \text{ to MCP} \\ 0 & N \end{cases}$$

$$P(Y=1 \mid X=1) = P(Y \mid F)$$

gender	MCP Y ($Y=1$)	N ($Y=0$)	
F ($X=1$)	29	20	49
M ($X=0$)	52	5	57
	81	25	106

$$= \frac{29}{49} = \frac{P(Y=1 \text{ and } X=1)}{P(X=1)} = \frac{\frac{29}{\cancel{106}}}{\frac{49}{\cancel{106}}}$$

(10.54)

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$$

$$(Y_i \mid \theta) \overset{IID}{\sim} \text{Bernoulli}(\theta)$$