① 07/31/19

Lecture 2 AMS 131

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

if $P(A \text{ and } B) = 0$, A and B are mutually exclusive

Ⓐ	not A

S: sample space

$0 \le P(A) \le 1$

"impossibility" "certainty"

False True

$P(S) = 1$

$P(S) = P(A \text{ or not } A)$ $1 = P(A) + P(\text{not } A)$

difficult to directly find → $P(A) = 1 - P(\text{not } A)$ ← indirect, easier

AND

pop.: 1, 2, 7 → sample: Y_1 [y_1] ← value, Y_2 [y_2] $n = 2$

↑ random variable (capital letter)

$P(Y_1 = 7 \text{ and } Y_2 = 7)$

replacement would be nice as there is only one 7!

case 1: at random with replacement

↳ IID independent identically distributed

$S = \{(1,1), (1,2), (1,7) \ldots (7,7)\}$ all outcomes

	1	2	7
1	(1,1)	(1,2)	(1,7)
2	(2,1)	(2,2)	(2,7)
7	(7,1)	(7,2)	(7,7)

3x3 contingency table

ELM? yes, by IID

$P(Y_1 = 7 \text{ and } Y_2 = 7) = \frac{1}{9}$

$P(Y_1 = 7) = \frac{3}{9} = \frac{1}{3} = P(Y_2 = 7) = \frac{3}{9} = \frac{1}{3}$

AMS131 Lecture 2 (cont.)

$P(Y_1 = 7 \text{ and } Y_2 = 7) = P(Y_1 = 7) \cdot P(Y_2 = 7)$

Conjecture: $P(A \text{ and } B) = P(A) \cdot P(B)$

Case 2: at random without replacement
↳ SRS simple random sampling

$P_{SRS}(Y_1 = 7 \text{ and } Y_2 = 7) = 0$ (common sense)

	1	2	7
1	~~(1,1)~~	(1,2)	(1,7)
2	(2,1)	~~(2,2)~~	(2,7)
7	(7,1)	(7,2)	~~(7,7)~~

if you know the first result, the probability changes – if you don't (i.e. take one, then just take another then ELM still applies

ELM? yes, by SRS

$P_{SRS}(Y_1 = 7) = \frac{2}{6} = \frac{1}{3}$ one sample at a time:
$P_{SRS}(Y_2 = 7) = \frac{2}{6} = \frac{1}{3}$ marginal behavior

What is the chance both are 7? It doesn't matter if the first is 7 – under ELM, we don't know the first result, they all have equal chance.

$P_{SRS}(Y_1 = 7 \text{ and } Y_2 = 7) = 0 \neq P_{SRS}(Y_1 = 7) = \frac{1}{3} \cdot P_{SRS}(Y_2 = 7) = \frac{1}{3}$

This leads to conditional probability
(Abraham de Moivre + Thomas Bayes)
1710 1750

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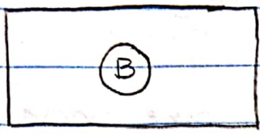
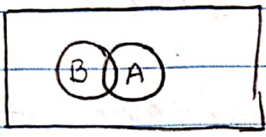
$P(B \text{ given } A) = P(B|A)$ "given"



$P(B) = \frac{B}{\square} = 1.$ area representation

← dart has to hit



$P(B|A) = \frac{B\,A}{A}$ ← we know it fell in A.

$P(B|A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}$ if $P(A) > 0$

undef. if $P(A) = 0$

if $P(A) > 0$ $P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$

if $P(B) > 0$ $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

$P(B \text{ and } A) = P(B|A)\,P(A)$ general product rule

$P(A \text{ and } B) = P(A|B)\,P(B)$ for AND

$P_{SRS}(Y_1 = 7 \text{ and } Y_2 = 7)$

$= P_{SRS}(Y_1 = 7) \cdot P_{SRS}(Y_2 = 7 \mid Y_1 = 7)$

$= \frac{1}{3} \cdot 0 = 0$ ✓

$P_{IID}(Y_1 = 7 \text{ and } Y_2 = 7)$

$= P_{IID}(Y_1 = 7) \cdot P_{IID}(Y_2 = 7 \mid Y_1 = 7)$

with at random with replacement (IID), the second draw doesn't change based on the first (independent of each other).

Bayesian ↔ how much information you have.

A and B are independent if information about B doesn't change P(A) and vice versa.

AMS 131 Lecture 2 (cont.)

Frequentist: A and B are independent if and only if $P(B|A) = P(B)$ and $P(A|B) = P(A)$

- after A (or B), and if they are independent, one should not influence the other

moreover, A and B are independent iff

$P(A \text{ and } B) = P(A) \cdot P(B|A)$
$= P(A) \cdot P(B)$

$P(B \text{ and } A) = P(B) \cdot P(A|B)$
$= P(B) \cdot P(A)$

special case of product rule under independence:
A, B independent iff $P(A \text{ and } B) = P(A) \cdot P(B)$

T-S case study

P(one or more T-S babies in a family of 5 kids, both parents are carriers)

* extra problems in book (answers to odd #'s)

P(one or more) = 1 − P(0 T-S babies)

= 1 − P(not T-S on 1ST and not T-S on 2ND AND .... not T-S on 5TH)

independence part of IID = $1 - \left[P\left(\frac{\text{not T-S}}{\text{1ST}}\right) \cdot P\left(\frac{\text{not T-S}}{\text{2ND}}\right) \cdots P\left(\frac{\text{not T-S}}{\text{5TH}}\right)\right]$

$= 1 - (1-\frac{1}{4})(1-\frac{1}{4}) \ldots (1-\frac{1}{4})$ identically distributed

P(one or more) $= 1 - (1-\frac{1}{4})^5 \doteq 76\%$

P(1 or more bad things occurring in n independent occurrances, P(bad thing) = p)
$= 1 - (1-p)^n$ special case of binomial probability

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Odds 'o' in favor of A (true statement)

$o = \frac{p}{1-p}$ $p = P(A)$ ratio of A happening vs. not happening

odds of 1+ T-S babies $= \frac{0.76}{1-0.76} = \frac{0.76}{0.24} \doteq 3$

solve for p $o = \frac{p}{1-p}$ $o(1-p) = p$ $o - op = p$

$p + op = o$ $p(1+o) = o$

$p = \frac{o}{1+o}$ ← always, 'o' can't be neg.

EXTRA NOTES (most important from book)

An experiment $\mathcal{E}$ is a data generating process in which all possible outcomes can be listed before $\mathcal{E}$ is performed

An event E is a set of possible outcomes of an experiment $\mathcal{E}$.

$\mathcal{E}$: process of having 5 kids, T-S or not

E: {at least 1 T-S baby}

The sample space S is the set of all possible outcomes of an experiment $\mathcal{E}$ (elements)

T = (T-S baby) N = (not T-S baby)

S = {NNNNN, ..., TTTTT}

no interest

↳ NNNNN; NTNNN etc.

TNNNN NNTNN ; E

the number of elements:

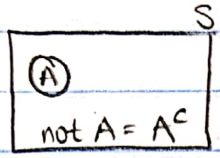
2 possibilities (T/N)

5 babies $= 2^5 = 32$ outcomes

AMS 131 Lecture 2 (cont.)

S is a product space

$\{T,N\} \times \{T,N\} \overset{\times}{\ldots} \{T,N\} = \{T,N\}^5$



dart must land inside sample space, either in A or A^c

$s \in S$: s is an element of sample space S

Set A is contained in Set B $(B \supset A)$

A is a subset of B if every element in A is in set B.

If A and B are events

$A \subset B \underset{\text{(iff)}}{\leftrightarrow}$ A occurs then so does B

Consequences theorem: If A, B, C are events then: (a) $A \subset B$ and $B \subset A \leftrightarrow A = B$

(b) $A \subset B$ and $B \subset C \leftrightarrow A \subset C$

cardinality of set A, $|A|$, is the number of distinct elements in A

ex: T-S case $|S| = 32$ (see page 5)

The set of all subsets is called the power set of the initial set, denoted 2^S

if $|S| = n$ cardinality

then $|2^S| = 2^n$ power set cardinality

$S = \{0, 1\}$

$\emptyset$ (null set)

$\{0\}$

$\{1\}$

$\{0, 1\}$

$|S| = 2$

$|2^S| = 4 = 2^2$

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If $S = \{a, b, c\}$ $|S| = 3$ $|2^3| = 8$

$\emptyset$ $\{a\}$ $\{b\}$ $\{c\}$ $\{a,b\}$ $\{b,c\}$ $\{a,c\}$ $\{a,b,c\}$

- enummerating power set = Pascal's Triangle

If $|S|$ is finite = good

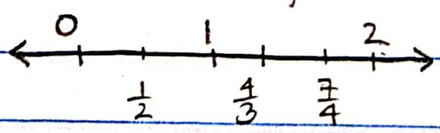
$|S|$ is infinite = bad (infinite set)

If the elements of an infinite set can be placed in a 1-to-1 correspondence with positive integers $\mathbb{N} = \{1, 2, 3 \ldots\}$, set A is <u>countably infinite</u>

$\updownarrow$ same order of infinity

$\{0, 1, 2, 3 \ldots\}$

rational numbers: real numbers expressed as ratio of integers (ex: $\frac{1}{2}$, $-\frac{89}{200}$ ...)



same number of rational numbers to integers

rational + irrational numbers = uncountable